Hybrid quantum computing problems and algorithms

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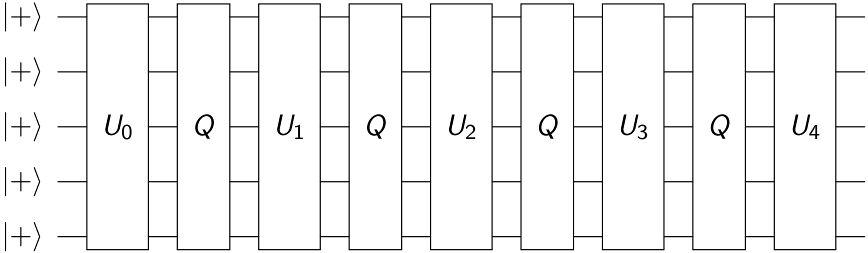
Quantum query algorithm 1

- $x_1 x_2 \ldots x_N \in \{0,1\}^N$
- $|\phi_{start}\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle |0\ldots0\rangle$
- Linear algebra: $U_t Q U_{t-1} Q \ldots . U_1 Q U_0 |\phi_{start}\rangle$
- Circuit:



Quantum query algorithm 2

- U_i - unitary
- $Q\,|i\rangle = (-1)^{x_i}\,|i\rangle$
- Complexity: number of Q

Hybrid Quantum query algorithms

Hybrid Quantum query algorithm

Classical algorithm that can call a quantum algorithm, which spends $\leq q$ queries.
Complexity is the total number of queries algorithm has used in the worst case.

Hybrid Quantum query algorithms - motivation

- Current quantum computers are not stable - one of the problems is the maximum circuit depth.
- Cannot run Grover's search: $O(\sqrt{N})$ and find solutions for many other problems.
- **!! 85%* of all algorithms have a Grover-like component !!**
- $\implies$ Natural restriction is the circuit depth: q.
- How does this affect complexity of problems?

Hybrid Quantum query algorithms - results (Other authors)

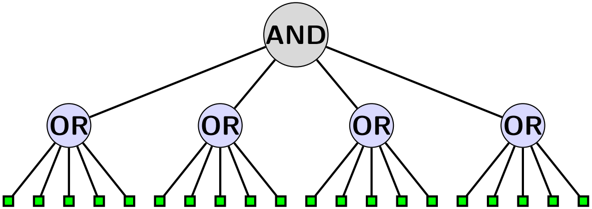
Search

We can find marked elements in an unstructured list with a hybrid quantum algorithm using - $\Theta(\frac{N}{q} + \sqrt{N})$ queries ([Sun and Zheng., 2019])

Collision lower bounds

Lower bounds for the collision finding problem is $\Omega(\sqrt{\frac{N}{q}} + N^{1/3})$ ([Hamoudi et al., 2024]).

Hybrid Quantum query algorithms: AND-OR



AND-OR

We can solve $AND_N \circ OR_M$ problem in the hybrid setting using:

- $O(\frac{NM}{q} + \sqrt{NM})$ queries, if $q \geq \sqrt{M}$.
- $O(\frac{NM}{q}\sqrt{\log N} + \sqrt{NM})$, else.

Lower bounds: $\Omega(\frac{NM}{q} + \sqrt{NM})$ **???**

Hybrid Quantum query algorithms: Collision

Definition of the Collision problem

Collision problem - we are given N numbers and a promise, that either all are different or every number have 2 copies. For example: $\{101, 23, 89, 12, 42, 97\}$ or $\{101, 97, 23, 23, 101, 97\}$. We need to decide which one of the two cases we have.

Collision

We can solve collision problem in the hybrid setting using $\Theta(\sqrt{\frac{N}{q}} + N^{1/3})$ queries. *Follows from a result of our group and [Hamoudi et al., 2024].*

Hybrid Quantum query algorithms: Distinguish k and k+1

Definition of k vs k+1 problem

We are given N elements and a promise that either k of them are marked or $k+1$. We need to distinguish both cases.

k vs k+1

We have the following 2 algorithm that are based on combinatorial proofs:

- We can solve this problem in the hybrid setting by using $O(\frac{N}{q}\sqrt{\frac{\log N}{\log\log N}} + \sqrt{Nk})$.
- There is also $O(\frac{N}{q}\sqrt{\frac{1}{\varepsilon}} + \sqrt{Nk})$ algorithm, where $k \leq N^{1-\varepsilon}$.

We have proven lower bounds $\Omega(\frac{N}{q} + \sqrt{Nk})$. We also have hypothesis that there is an $O(\frac{N}{q}\sqrt{\log\log N})$ algorithm (complicated to prove).

High probability Grover's search

Reduced depth ε-Grover

Another topic of our interest was Grover's search with with error ε. There is an algorithm of complexity $O(\sqrt{N \log \frac{1}{\varepsilon}})$ and depth $\in O(\sqrt{N})$. Can we reduce the depth without sacrificing overall complexity? Maybe do sacrifice something, but get a favorable tradeoff?

Small quantum circuits on Quantinuum hardware

- Work with quantum hardware using Qiskit, Pytket libraries.
- Testing simple yet essential circuits on hardware.
- Results - to be determined....

Further work

- Improve upper bounds for k vs k+1
- Lower bounds? (*Very complicated...*)
- Another computing model?
- Change of topic?

References

Hamoudi, Y., Liu, Q., and Sinha, M. (2024).
The nisq complexity of collision finding.
Springer Nature Switzerland, pages 3–32.

Sun, X. and Zheng., Y. (2019).
Hybrid decision trees: Longer quantum time is strictly more powerful.
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