Hybrid quantum computing problems and algorithms

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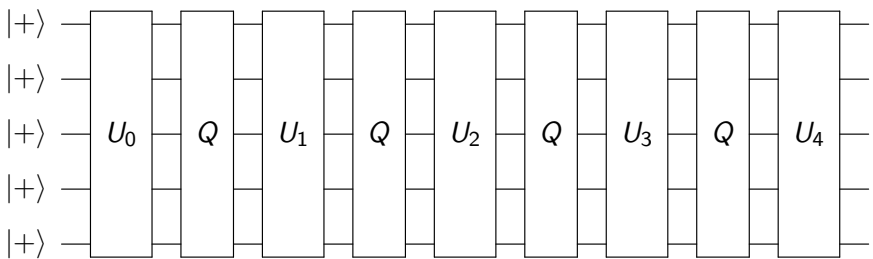
February 13, 2025

Quantum query algorithm 1

- $x_1 x_2 ... x_N \in \{0,1\}^N$
- $|\phi_{start}\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |i\rangle |0...0\rangle$
- Linear algebra: $U_t Q U_{t-1} Q .... U_1 Q U_0 |\phi_{start}\rangle$
- Circuit:



Quantum query algorithm 2

- U_i - unitary
- $Q|i\rangle = (-1)^{x_i}|i\rangle$
- Complexity: number of Q

Hybrid Quantum query algorithms

Hybrid Quantum query algorithm

Classical algorithm that can call a quantum algorithm, which spends $\leq q$ queries.
Complexity is the total number of queries algorithm has used in the worst case.

Hybrid Quantum query algorithms - motivation

- Current quantum computers are not stable - one of the problems is the maximum circuit depth.
- Cannot run Grover's search: $O(\sqrt{N})$ and solutions for many other problems.
- $\Longrightarrow$ Natural restriction is the circuit depth: q.
- How does this affect complexity of problems?

Hybrid Quantum query algorithms - results

Search

We can find marked elements in an unstructured list with a hybrid quantum algorithm using - $\Theta(\frac{N}{q} + \sqrt{N})$ queries ([Sun and Zheng., 2019])

Hybrid Quantum query algorithms - results before

- $AND \circ OR$: $\tilde{O}(\frac{N*M}{q} + \sqrt{N*M})$
- Collision: $O(\sqrt{\frac{N}{q}} + N^{1/3})$
- k vs k+1: $\tilde{O}(\frac{N}{q} + \sqrt{N*k}), \Omega(\frac{N}{q} + \sqrt{N*k})$

Hybrid Quantum query algorithms - results NEW I

Search with small error

We can find a marked element in a unstructured list with a hybrid quantum query algorithm with error $\epsilon > 0$ spending $\Theta(\frac{N}{q} + \sqrt{N * \log(1/\epsilon)})$ queries. From this comes the algorithm for $AND \circ OR$: $O(\frac{N*M}{q} + \sqrt{N * M * \log 1/M})$.

Hybrid Quantum query algorithms - results NEW II

Distinguish k and k+1

We can distinguish between k and k+1 marked elements with a hybrid quantum query algorithm using no more than $min(O(\frac{N}{q} + \sqrt{N * k * \log k}), O(\frac{N}{q} * \sqrt{\frac{\log k}{\log \log k}}))$ queries.

Further work

- Improve upper bounds for k vs k+1
- Some non trivial results for element distinctness problem
- Implementation of a toy example on a real device
- Lower bounds? (*Very complicated...*)
- Another computing model?

References


Sun, X. and Zheng., Y. (2019).
Hybrid decision trees: Longer quantum time is strictly more powerful.
arXiv:1911.13091.

The End