Quantum Circuit Optimization

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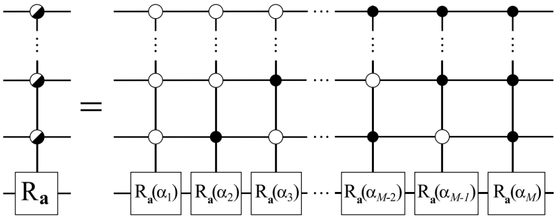
Unitary Decomposition

$$\begin{pmatrix} R_0 & \overline{0} & \dots & \overline{0} \\ \overline{0} & R_1 & \dots & \overline{0} \\ \dots & \dots & \dots & \dots \\ \overline{0} & \overline{0} & \dots & R_{2^n-1} \end{pmatrix} \rightarrow$$ **A circuit consisting of a set of basic gates**

Uniformly Controlled Rotation Operation

- Given *n* control qubits and one target qubit
- $R_y(\alpha_i)$ - rotation of a qubit around y-axis by an angle α_i
- $UC^nR(\alpha)$, where $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_M\}$ and $M = 2^n$

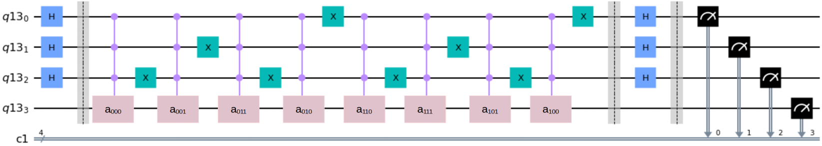
Figure



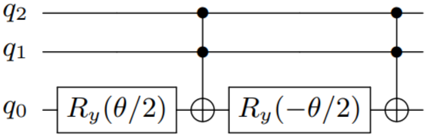
Application of $UC^nR(\alpha)$ operation

- Implementation of a general unitary transformation decomposition (Mottonen et.al., 2004)
- A general state preparation (Mottonen et.al., 2004)
- Algorithms based on fingerprinting technique (QFA, quantum hashing algorithms, etc.) (Frievalds, Ambainiz)

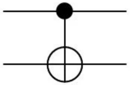
$UC^nR(\alpha)$ implementation

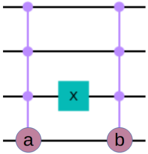


controlled $R_y(\theta)$ naive decomposition



CNOT-cost of the circuit



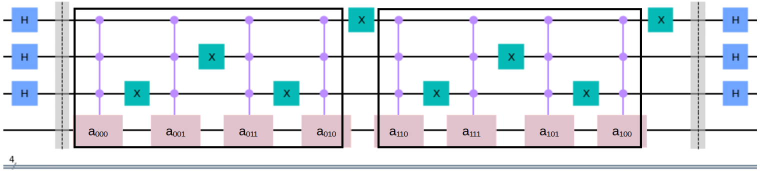


A naive circuit costs

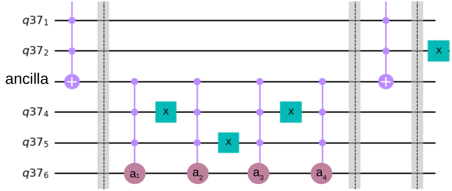
- $S_2(n) = 2 \cdot \$(C^n R(\theta)) = 4\$(C^n X)$.
- $S_2(n) = 192(n-3)$ if $n > 4$ and $\$(C^n X) = 48(n-3)$
- $S_2(n) = 96(n-2)$ if $n > 4$ and $\$(C^n X) = 24(n-2)$.

$S_2(2) = 4\$(C^2 X) = 24$
$S_2(3) = 4\$(C^3 X) = 56$
$S_2(4) = 4\$(C^4 X) = 72$

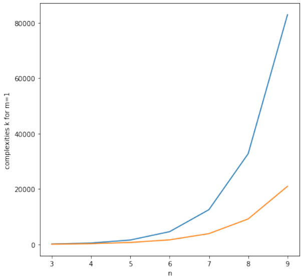
Circuit optimization. Ancilla qubit



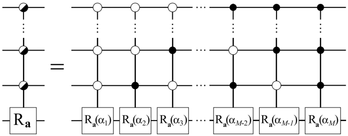
Ancilla adding :

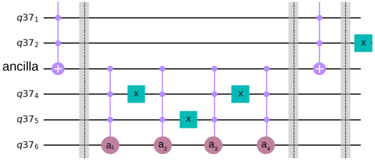


Circuit optimization. Ancilla qubit



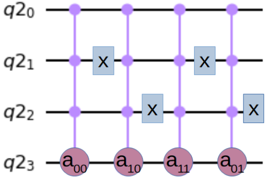
$$\Sigma_{new}(n) << \Sigma_{old}(n)$$



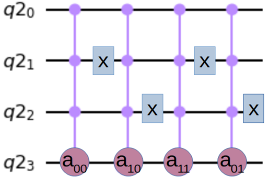


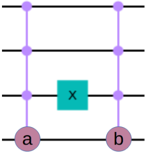
$\otimes 2^{n-k}$

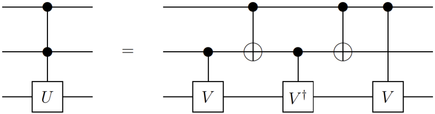
Circuit optimization.Subcircuit decomposition



Circuit optimization.Subcircuit decomposition



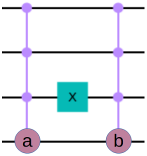


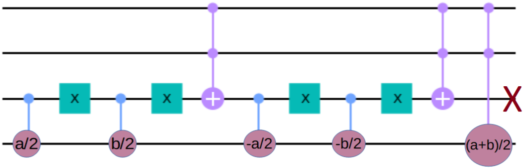


U is rotation on some γ and V is rotation on $\gamma/2$
$U = V^2$

Circuit optimization.Subcircuit decomposition without extra memory

For 2 angles



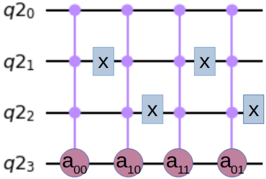


n	$S_2(n)$	$S_2^{naive}(n)$
2	12	24
3	32	56
4	64	72
n	$192n - 760$	$192n - 576$

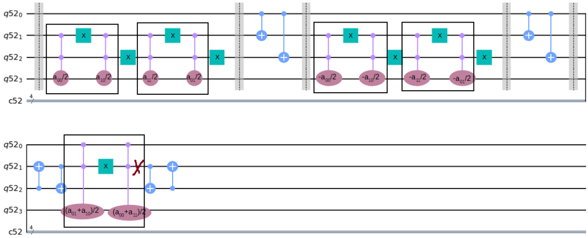
Table – Cnot-cost of naive and optimized circuits for a pair of n-controlled rotations

For 4 angles

$S_4^{naive}(n) = 384n - 1152$



$S_4(n) = 384n - 1860$



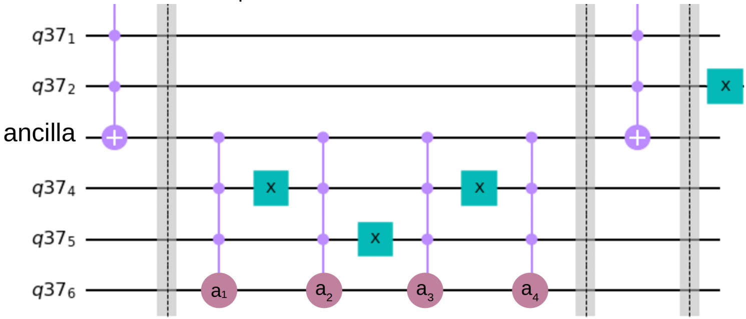
Circuit optimization.Subcircuit decomposition

g (2^g angles)	first met.	divid.met.	naive circuit
1	$192n - 760$	-	$192n - 576$
2	$384n - 1860$	$384n - 1520$	$384n - 1152$
3	$672n - 3692$	$768n - 3040$	$768n - 2304$

Table – cnot-costs $S_{2g}(n)$ of circuits implementing $UC^nR(\Theta)$, where $|\Theta| = 2^g$

Circuit optimization

Uniformly controlled rotation of the target qubit with *n* controllers.
k is a number of "active" qubits.



$\times 2^{n-k}$

Rotation on 2^g angles with n controllers

$S_g(n)$ is a complexity of uniformly control rotation of target qubit on 2^g angles with n control qubits.
Results :

- If $g > 1.7\log(n)$, then
  $S_g(n) = 2 \cdot S_{g-1}(n)$
- If $g < 1.7\log(n)$, then
  $S_g(n) = 4 \cdot S_{g-1}(g) + 2gS_{cx}(n-g) + S_{g-1}(n-1) + 4(g-1)$

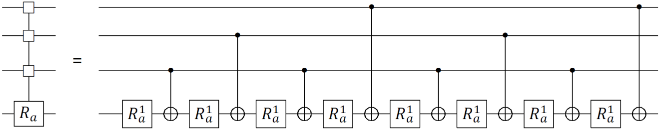
naive	with ancilla	without extra memory	combination
$2^n \cdot 96(n-3)$	$2^n \cdot 48(1 + \log n)$	$2^n \cdot 110\sqrt{n}$	$2^n \cdot 110\sqrt{\log n + 1}$

Table – cnot-costs of different circuits which implement $UC^nR(\Theta)$

Result which we found further

Mottonen et.al. proposed

Figure – The circuit of uniformly controlled rotation with 3 control qubits.



Our nearest plan is to

- Optimize circuits for different topologies of qubits in different quantum accelerators.
- Optimize circuit for arbitrary gates located between multi-controlled rotation gates
- Publish result

Thank you !