The background features a dark blue gradient with a starry space pattern. Overlaid on this are several technical diagrams, including circular gauges with numerical scales (e.g., 140, 150, 160, 170, 180, 190, 200, 210, 220, 230, 240, 250, 260) and various circular and curved lines, some with arrows indicating direction. The overall aesthetic is futuristic and scientific.

“HYBRID QUANTUM COMPUTING PROBLEMS AND ALGORITHMS”

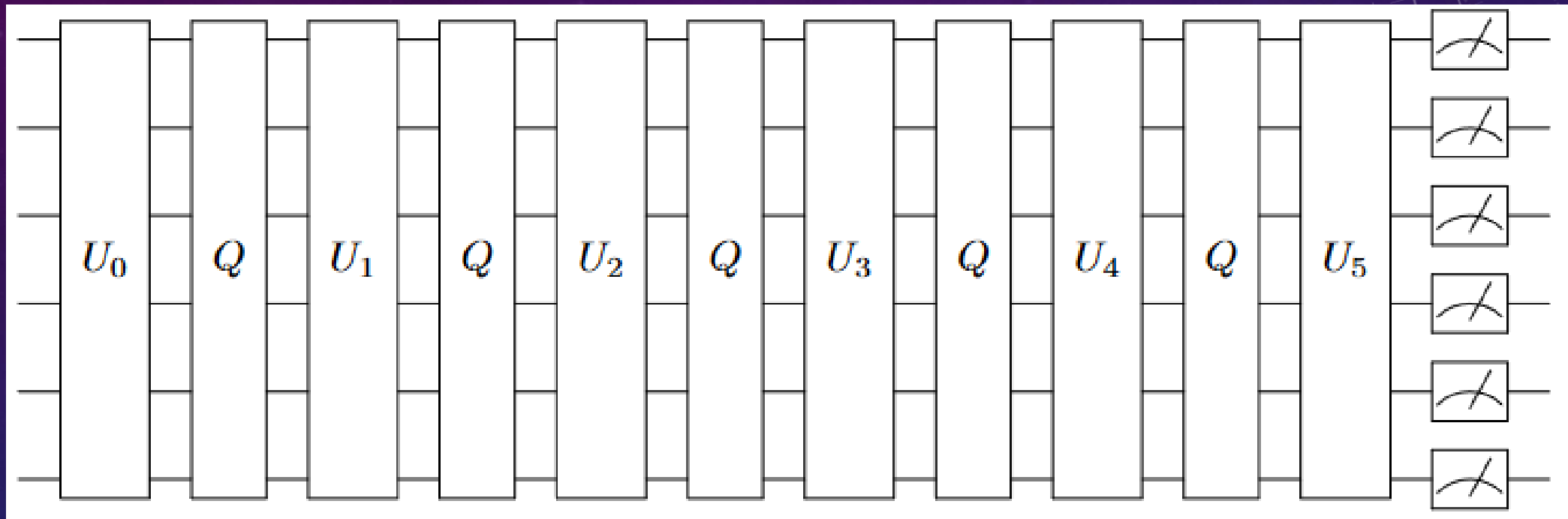
ANSIS ZVIRBULIS, 2ND YEAR

SUPERVISOR: PROF., DR. SC. COMP. ANDRIS AMBAINIS

QUANTUM COMPUTING QUERY MODEL

- Quantum state: $|\psi\rangle$
- Unitary transformations: U_0, U_1, \dots, U_T
- Quantum query transformation: Q on input x_1, x_2, \dots, x_N :
 - $Q |x_i\rangle \rightarrow (-1)^{x_i} |x_i\rangle$
 - $Q |x_i\rangle |a\rangle \rightarrow |x_i\rangle |a \oplus x_i\rangle$
- $|\psi_{end}\rangle = U_T Q U_{T-1} Q \dots U_1 Q U_0 |\psi_{start}\rangle$
- U_i - doesn't depend on input
- Counting the number of queries

CIRCUIT EXAMPLE



$$q = 5$$

QUANTUM COMPUTING: ALGORITHMS

- Grover's search: $\Theta\left(\sqrt{\frac{n}{k}}\right)$
- Element distinctness: $\Theta\left(n^{\frac{2}{3}}\right)$

$Q(f, q)$

- $Q(f)$ but restriction of max q queries
- $Q(f) \leq Q(f, q)$, but how much?
- $Q(OR) = \sqrt{N}, Q(OR, q) = \Theta\left(\frac{N}{q}\right)$

$Q(f, q)$ - IMPORTANCE

Depth of circuit (# of operations) – serious challenge (likely for a long time in future)

- Noise
- Instability

RESULTS: $Q(\text{collision}, q)$

$$O\left(\frac{n}{q^2} + \sqrt[3]{n}\right) \Rightarrow O\left(\sqrt{\frac{n}{q}}\right)$$



RESULTS: $Q(ED, q)$



$$Q(ED, q) = O\left(\frac{N^2}{q^2}\right)$$



RESULTS: $Q(AND \circ OR; q)$



$$O\left(\frac{N * M}{q}\right) \Rightarrow \mathbb{I}\left(\frac{N * M}{q}\right)$$



RESULTS: $Q(k \text{ vs } k + 1, q)$

$$\Omega\left(\frac{n}{q} + \sqrt{n * k}\right)$$

$$\tilde{O}\left(\frac{n}{q} + \sqrt{n * k}\right)$$

RESULTS: $Q(\text{AND} - \text{OR tree}, q)$

$$\Omega\left(\frac{N}{q} + \sqrt{N}\right)$$

Regular with constant depth: $\Theta\left(\frac{N}{q} + \sqrt{N}\right)$

Non-regular: $O\left(\frac{N}{q} * \log^t N\right)$

2 2
@ @

$t=2$
@

check

RESULTS – LOWER BOUNDS

New and more universal approach – very hard to find!!!

FUTURE

- Finalize AND-OR trees
- Lower bounds
- Element distinctness / quantum walk
- Montenegro algorithm for CSP
- Other problems???

- **Implementation on real quantum computer**



THANK YOU!