“HYBRID QUANTUM COMPUTING PROBLEMS AND ALGORITHMS”

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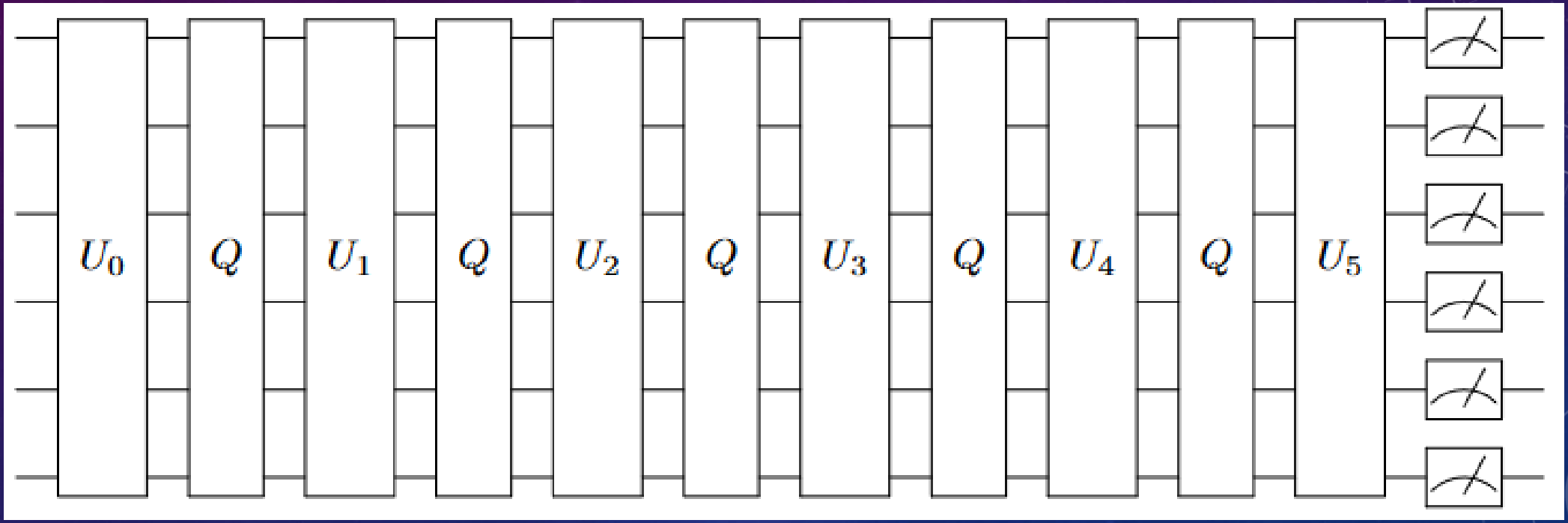
QUANTUM COMPUTING QUERY MODEL

- Quantum state: $|\psi\rangle$
- Unitary transformations: $U_0, U_1, \dots, U_T$
- Quantum query transformation: Q on input $x_1, x_2, \dots, x_N$:
  - $Q\ |x_i\rangle \rightarrow (-1)^{x_i}|x_i\rangle$
  - $Q\ |x_i\rangle\ |a\rangle \rightarrow |x_i\rangle|a\ \oplus\ x_i\rangle$
- $|\psi_{end}\rangle = U_T Q U_{T-1} Q \dots U_1 Q U_0 |\psi_{start}\rangle$
- U_i - doesn't depend on input
- Counting the number of queries

CIRCUIT EXAMPLE



$q = 5$

QUANTUM COMPUTING: ALGORITHMS

- Grover's search: $\Theta\left(\sqrt{\frac{n}{k}}\right)$
- Element distinctness: $\Theta\left(n^{\frac{2}{3}}\right)$

$Q(f, q)$

- $Q(f)$ but restrisctrion of max q queries
- $Q(f) \leq Q(f, q)$, but how much?
- $Q(OR) = \sqrt{N}, Q(OR, q) = \Theta\left(\frac{N}{q}\right)$

$Q(f, q)$ - IMPORTANCE

Depth of circuit (# of operations) – serious challenge (likely for a long time in future)

- Noise
- Instability

RESULTS: $Q(collision, q)$

$$O\left(\frac{n}{q^2} + \sqrt[3]{n}\right) \Rightarrow O\left(\sqrt{\frac{n}{q}}\right)$$

RESULTS: $Q(ED, q)$

$$Q(ED, q) = O\left(\frac{N^2}{q^2}\right)$$

RESULTS: $\mathrm{Q}(AND \circ OR; q)$

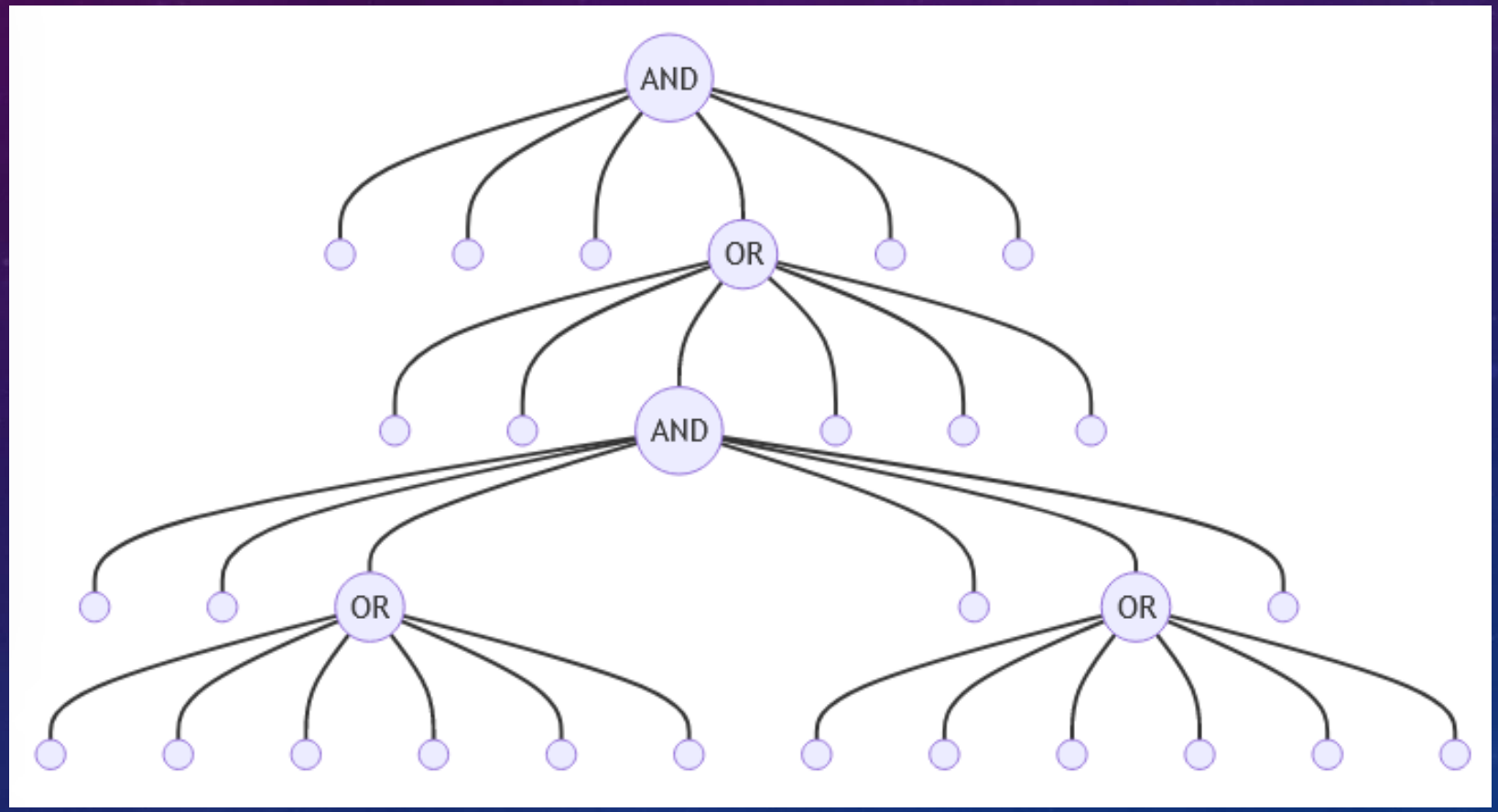
$$O\left(\frac{N * M}{q}\right) \Longrightarrow \Theta\left(\frac{N * M}{q}\right)$$

RESULTS: $Q(k\ vs\ k+1, q)$

$$\Omega\left(\frac{n}{q}+\sqrt{n*k}\right)$$

$$\tilde{O}\left(\frac{n}{q}+\sqrt{n*k}\right)$$

RESULTS: $Q(AND - OR\ tree, q)$



RESULTS: $Q(AND - OR\ tree, q)$

$\Omega\left(\frac{N}{q} + \sqrt{N}\right)$

Regular with constant depth: $\Theta\left(\frac{N}{q} + \sqrt{N}\right)$

Non-regular: $O\left(\frac{N}{q} * \log^t N\right)$

Check

$t = 2$

RESULTS – LOWER BOUNDS

New and more universal approach – very hard to find!!!

FUTURE

- Finalize AND-OR trees
- Lower bounds
- Element distinctness / quantum walk
- Montenaro algorithm for CSP
- Other problems???

- **Implementation on real quantum computer**

THANK YOU!